Section 6.1 – Law of SINES

So far, all the triangles we've solved have had one thing in common- they have all been ***right*** triangles. However, we can use sine and cosine to solve ***oblique*** triangles too - triangles WITHOUT a right angle.

To solve an oblique triangle, you must know the measure of at least one SIDE, and any two other parts of the triangle. The possibilities are:

1) ____________ 2) ____________ 3) ____________ 4) ____________ 5) ____________

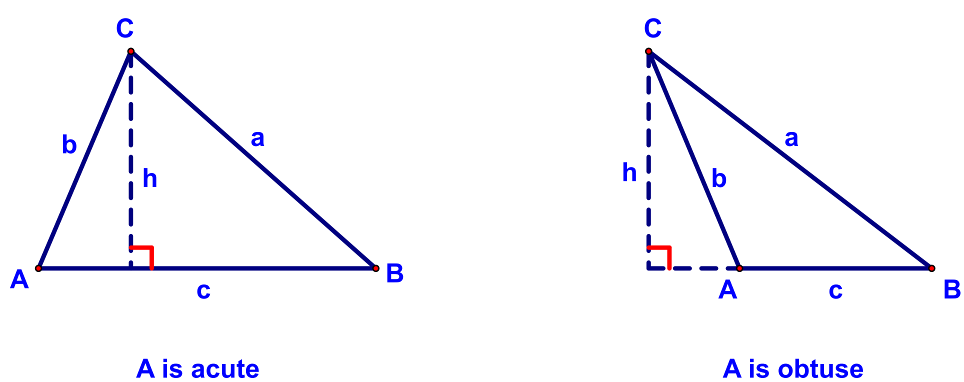
Only three of these situations can be solved with **Law of Sines** – the other two will use **Law of Cosines**. Today, we're going to discuss two of the first three.

Law of Sines

If $\triangle ABC$ has sides a, b, and c, then

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

The triangle will look like one of the two shown below:



The Law of Sines can also be written in reciprocal form:

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$

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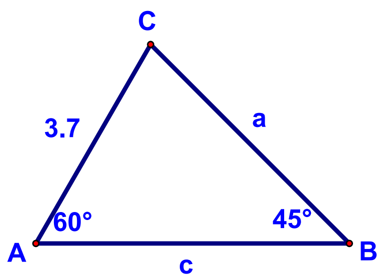
The AAS Case:

For the triangles below, find the remaining sides and angles

1)

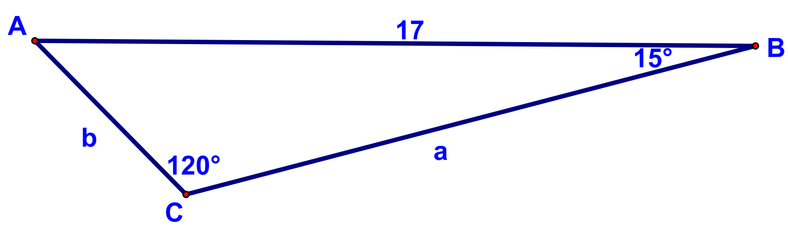


m∠C =

a =

c =

2)



m∠A =

a =

b =

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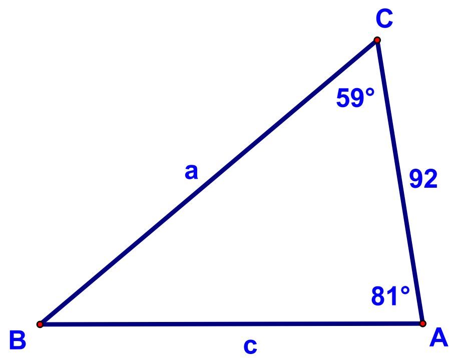
The ASA case:

3)



$m\angle B =$

$a =$

$c =$

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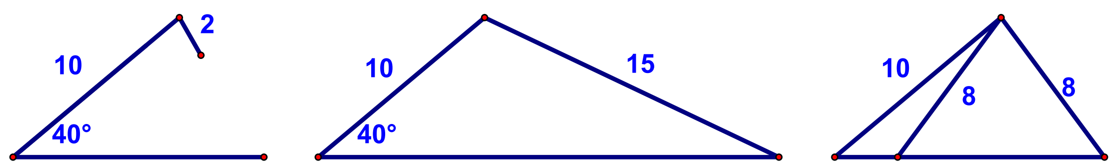
The SSA case (the ambiguous case):

Why is this ambiguous?
In Geometry, you learned that you could prove that two triangles were congruent using the following methods:

However, when you were given two sides and the NON-included angle (SSA) then, depending on the information given, you could construct ____, ____, or ___ triangles.
Here is what they look like:



0 Triangles 1 Triangle 2 Triangles

So this means that **not just one** unique triangle can necessarily be created.

How do we figure out if there are 0, 1, or 2 triangles with a SSA problem?
Draw a triangle with $m\angle A = 30°,\ b = 10$

What do we know about side "a"?

Now: $If\ \ a < 5 \rightarrow$
$If\ \ a > 10 \rightarrow$
$If\ \ 5 < a < 10 \rightarrow$

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Ex. 1) In ΔABC, $m\angle A = 30°$, $a = 7$, and $b = 12$.
Solve the triangle for all the missing sides and angles:

Ex. 2) In ΔABC, $m\angle A = 30°$, $a = 4$, and $b = 12$.
Solve the triangle for all the missing sides and angles:

Ex.3) In ΔABC, $m\angle A = 20°$, $a = 12$, and $b = 10$.
Solve the triangle for all the missing sides and angles:

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Ex.4) In ΔABC, $m\angle A = 30°$, $a = 6$, and $b = 12$.
Solve the triangle for all the missing sides and angles:

Ex.5) In ΔABC, $m\angle A = 40°$, $a = 75$, and $b = 85$.
Solve the triangle for all the missing sides and angles:

Ex.6) In ΔABC, $m\angle A = 85°$, $a = 15$, and $b = 25$.
Solve the triangle for all the missing sides and angles:

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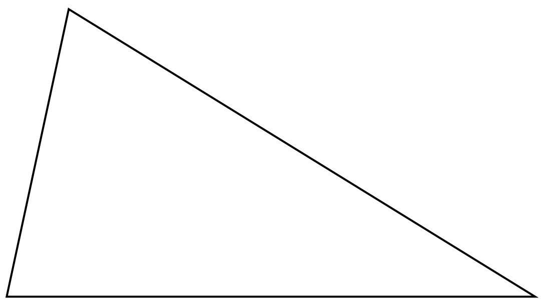
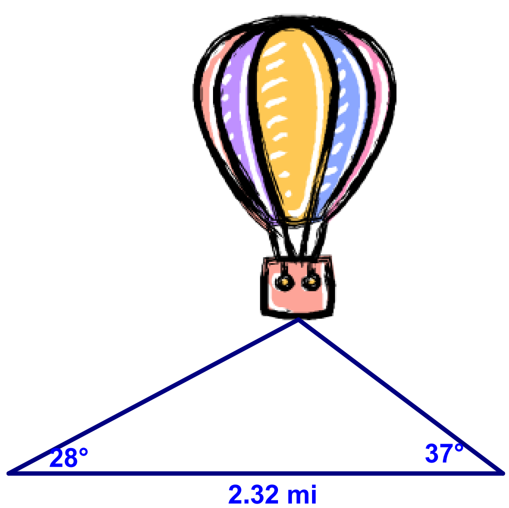
Law of Sines: Applications!

1) A telephone pole tilts AWAY from the sun at a 7° angle from the vertical, and it casts a 27-foot shadow. The angle of elevation from the tip of the shadow to the top of the pole is 52°. How tall is the pole?

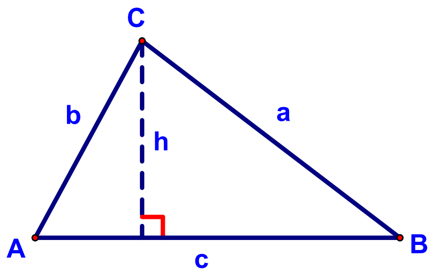


2) Observers 2.32 miles apart see a hot-air balloon directly between them but at the angles of elevation shown in the figure. Find the altitude of the balloon:



Section 6.1 – Law of SINES

Area of an oblique triangle



The area of $\triangle ABC =$

Now, play around and see if you can get h in terms of the sides a, b, and c.

In terms of a, h =

In terms of b, h =

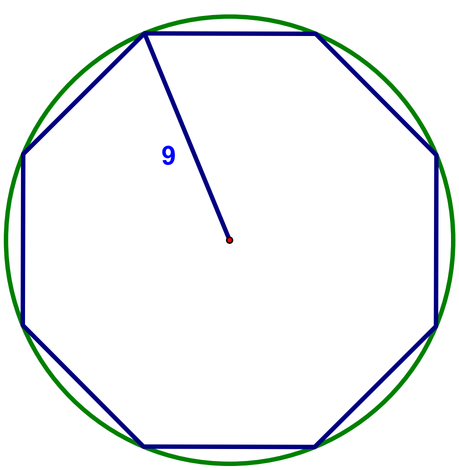
Therefore, using c as the base, the area of $\triangle ABC =$

Area of an Oblique Triangle

The area of any triangle is one-half the product of the lengths of the two sides times the sine of their included angle. That is,

$$\text{Area} = \frac{1}{2}bc\sin A = \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B$$

Example: Find the AREA of a regular octagon (equiangular and equilateral) inscribed in a circle of radius 9 inches:



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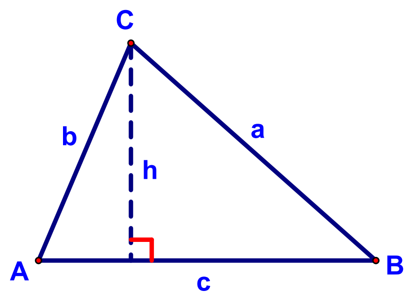
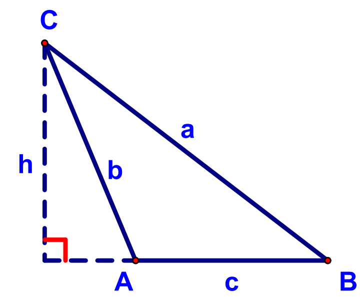
In case you are curious, there is a REASON why the Law of Sines works…

Proof



A is acute



A is obtuse

Let's see why the Law of Sines is true. Considering the triangles show above, you can see that

$\sin A = \frac{h}{b}$ or $h = b\sin A$, and

$\sin B = \frac{h}{a}$ or $h = a\sin B$

From this,

In a similar manner (you'd need an altitude from B to side $\overline{AC}$), you should be able to show that $\frac{c}{\sin C}$ equals the other two as well.